

# Influence of the Neutron Flux Characteristic Parameters in the Irradiation Channels of Reactor on NAA Results Using $k_0$ -Standardization Method

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**Abstract**—An approximation method using to estimate the influence of the uncertainties of the neutron flux characteristic parameters in the irradiation positions on the NAA results using  $k_0$ -standardization technique was presented. These parameters are the epithermal reactor neutron spectrum shape-factor  $\alpha$ , the effective resonance energy  $\bar{E}_r$  for a given nuclide and the thermal to epithermal neutron flux ratio  $f$ . The method is applied to estimate the effect of the uncertainties in the determination of  $\alpha$ ,  $\bar{E}_r$  and  $f$  on final NAA results for some irradiation channels of the Dalat reactor. The obtained results shows that presented method is suitable in practical use for the estimation of the errors due to the uncertainty of the neutron flux characteristic parameters at the irradiation position in reactor.

**Keyword**— $k_0$ -standardization Technique, Error Propagation Function, Neutron flux Characteristics, Dalat reactor

## I. INTRODUCTION

Since the  $k_0$ -standardization method was introduced in NAA [1], it has been broadly applied in the reactor in the world. The fundamental concept of  $k_0$ -method was being elaborated previously in great detail [1-3]. The concentration of a element in the  $k_0$ -method is calculated by:

$$\rho(\text{ppm}) = \frac{N_p/t_m}{\left(\frac{N_p}{\text{SDCW}}\right)^*} \frac{1}{k_0} \frac{f + Q_0^*(\alpha)}{f + Q_0(\alpha)} \frac{\varepsilon_p^*}{\varepsilon_p} \quad (1)$$

with  $k_0$  in Eq.(1) defined as:

$$k_0 = \frac{M^* \theta \sigma_0 \gamma}{M \theta^* \sigma_0^* \gamma^*} \quad (2)$$

In Eqs.(1) and (2):

$M$  - atomic mass;

$\theta$  - isotopic abundance;

$\sigma_0$  - 2200 m.s<sup>-1</sup> (n, $\gamma$ ) cross-section;

$\gamma$  - absolute gamma-intensity;

$N_p$  - peak area corrected for pulse losses;

$W$  - sample weight in gram;

$w^*$  - comparator weight in microgram;

$S = 1 - \exp(-\lambda t_{\text{irr}})$ ;  $t_{\text{irr}}$  - irradiation time;  $\lambda$  - decay constant;

$D = \exp(-\lambda t_d)$ ;  $t_d$  - decay time;

$C = [1 - \exp(-\lambda t_m)]/\lambda t_m$ ;  $t_m$  - measuring time;

$f$  - thermal to epithermal neutron flux ratio;

$Q_0(\alpha) = I_0(\alpha)/\sigma_0$ ;  $I_0(\alpha)$  - resonance integral corrected for a non-ideal epithermal neutron flux distribution (assumed  $1/E^{1+\alpha}$ );

$\varepsilon_p$  - detector's efficiency;

When the epithermal neutron flux distribution deviates from ideality, i.e. it does not follow the  $1/E$ -law,  $Q_0(\alpha)$  of nuclide  $i$  can be written by:

$$Q_{0i}(\alpha) = (Q_{0i} - 0.429) / (\bar{E}_{ri})^\alpha + 0.429 / [(2\alpha + 1)(0.55)^\alpha] \quad (3)$$

with  $\alpha$  - neutron spectrum shape factor deviating from the  $1/E$ -law, independent of neutron energy and  $|\alpha| \ll 1$ ;  $\bar{E}_{ri}$  - effective resonance energy of nuclide  $i$ ;

The asterisks in Eqs.(1) and (2) refers to the comparator, which is suitable for coirradiation with the sample; in most case, Au is used as a comparator. The  $k_0$ -factors to Au for interested isotopes in NAA were experimentally determined and tabulated in [4] with an accuracy better than 2% (average  $\sim 1\%$ ). The relevant nuclear data as  $Q_{0i}$  and  $\bar{E}_{ri}$  can be found in a tabulated form or in a computer library.  $\alpha$ ,  $f$  and  $\varepsilon_p$  must be determined by experiment and they depend on specific irradiation channel and detector, which are used in practice. The detector's efficiency ( $\varepsilon_p$ ) can be determined with an uncertainty about 2%; but the uncertainty of  $\alpha$  can be more than 10%, even bigger, depend on the irradiation channels in reactor. Since the term  $[f + Q_0^*(\alpha)]/[f + Q_0(\alpha)]$  in Eq. (1), it is clear that an additional parameter,  $\bar{E}_{ri}$ , should be considered, because the uncertainties of  $\bar{E}_{ri}$  of some nuclides are about 20% [4,5].

The accuracy and the applicability of the  $k_0$ -standardization method were detailedly presented in [5] by F. De CORTE et. al.. In [6], J.OP De BEEK evaluated the effect of errors of  $\alpha$  and  $\bar{E}_{ri}$  on the results in terms of concentration,

based on the  $^{197}\text{Au}$  comparator; in that  $Q_{0i}(\alpha)$  was approximated by :

$$Q_{0i}(\alpha) \approx Q_{0i}(\bar{E}_{ri})^{-\alpha} \quad (4)$$

However, with this approximation, it led that some results in [6] have to be put to discussion (see below).

In this work, we carry out an approximation method to evaluate the effect of errors of  $\alpha$  and  $\bar{E}_{ri}$  on the NAA results in the  $k_0$ - standardization method. The obtained results showed that the approximate method in this work is acceptable with confident accuracy.

TABLE I THE VALUES OF  $a_i$  FOR THE INTERESTING NUCLIDES IN NAA

Target Nuclide	Formed nuclide	$a_i$ with $\alpha < 0$	$a_i$ with $\alpha > 0$	Target Nuclide	Formed nuclide	$a_i$ with $\alpha < 0$	$a_i$ with $\alpha > 0$
$^{23}\text{Na}$	$^{24}\text{Na}$	0,524987	0,301907	$^{116}\text{Sn}$	$^{117m}\text{Sn}$	0,996203	0,992120
$^{26}\text{Mg}$	$^{27}\text{Mg}$	0,600414	0,260458	$^{122}\text{Sn}$	$^{123m}\text{Sn}$	0,958568	0,907080
$^{27}\text{Al}$	$^{28}\text{Al}$	0,632092	0,355905	$^{124}\text{Sn}$	$^{125m}\text{Sn}$	0,996581	0,993233
$^{37}\text{Cl}$	$^{38}\text{Cl}$	0,618735	0,340898	$^{121}\text{Sb}$	$^{122m}\text{Sb}$	0,99639	0,991949
$^{41}\text{K}$	$^{42}\text{K}$	0,744902	0,510677	$^{123}\text{Sb}$	$^{124m}\text{Sb}$	0,993951	0,988375
$^{45}\text{Sc}$	$^{46}\text{Sc}$	0,225177	0,13871	$^{127}\text{I}$	$^{128}\text{I}$	0,991934	0,984365
$^{50}\text{Ti}$	$^{51}\text{Ti}$	0,61391	0,300434	$^{133}\text{Cs}$	$^{134m}\text{Cs}$	0,993476	0,980868
$^{51}\text{V}$	$^{52}\text{V}$	0,484063	0,253348	$^{130}\text{Ba}$	$^{131m}\text{Ba}$	0,991752	0,983823
$^{50}\text{Cr}$	$^{51}\text{Cr}$	0,444713	0,235221	$^{132}\text{Ba}$	$^{133m}\text{Ba}$	0,961113	0,921759
$^{55}\text{Mn}$	$^{56}\text{Mn}$	0,768236	0,591296	$^{138}\text{Ba}$	$^{139}\text{Ba}$	0,721393	0,434897
$^{58}\text{Fe}$	$^{59}\text{Fe}$	0,744577	0,553371	$^{139}\text{La}$	$^{140}\text{La}$	0,824392	0,703294
$^{59}\text{Co}$	$^{60}\text{Co}$	0,899648	0,792867	$^{140}\text{Ce}$	$^{141}\text{Ce}$	0,696096	0,429943
$^{64}\text{Ni}$	$^{65}\text{Ni}$	0,603571	0,327784	$^{142}\text{Ce}$	$^{143}\text{Ce}$	0,798767	0,600323
$^{63}\text{Cu}$	$^{64}\text{Cu}$	0,786528	0,594356	$^{141}\text{Pr}$	$^{142m}\text{Pr}$	0,84513	0,710838
$^{65}\text{Cu}$	$^{66}\text{Cu}$	0,768558	0,578803	$^{146}\text{Nd}$	$^{147}\text{Nd}$	0,884097	0,750436
$^{64}\text{Zn}$	$^{65}\text{Zn}$	0,879728	0,716396	$^{148}\text{Nd}$	$^{149}\text{Nd}$	0,956282	0,908206
$^{68}\text{Zn}$	$^{69m}\text{Zn}$	0,928832	0,842012	$^{150}\text{Nd}$	$^{151}\text{Nd}$	0,982296	0,962764
$^{71}\text{Ga}$	$^{72}\text{Ga}$	0,967117	0,932900	$^{152}\text{Sm}$	$^{153}\text{Sm}$	0,995457	0,985053
$^{75}\text{As}$	$^{76}\text{As}$	0,984389	0,968602	$^{154}\text{Sm}$	$^{155}\text{Sm}$	0,949148	0,899154
$^{74}\text{Se}$	$^{75}\text{Se}$	0,982325	0,966392	$^{153}\text{Eu}$	$^{154m}\text{Eu}$	1,002410	0,972428
$^{79}\text{Br}$	$^{80m}\text{Br}$	0,984481	0,963835	$^{158}\text{Gd}$	$^{159}\text{Gd}$	0,993709	0,987888
$^{81}\text{Br}$	$^{82m}\text{Br}$	0,988802	0,976569	$^{160}\text{Gd}$	$^{161}\text{Gd}$	0,941083	0,869589
$^{85}\text{Rb}$	$^{86m}\text{Rb}$	0,985106	0,962061	$^{159}\text{Tb}$	$^{160}\text{Tb}$	0,991765	0,983428
$^{87}\text{Rb}$	$^{88}\text{Rb}$	0,990543	0,978106	$^{164}\text{Dy}$	$^{165m}\text{Dy}$	-0,59612	-0,13894
$^{84}\text{Sr}$	$^{85m}\text{Sr}$	0,984748	0,963917	$^{165}\text{Ho}$	$^{166}\text{Ho}$	0,989663	0,976313
$^{86}\text{Sr}$	$^{87m}\text{Sr}$	0,945262	0,87112	$^{170}\text{Er}$	$^{171}\text{Er}$	0,950799	0,903065
$^{89}\text{Y}$	$^{90m}\text{Y}$	0,963615	0,891243	$^{169}\text{Tm}$	$^{170}\text{Tm}$	1,004700	0,991756
$^{94}\text{Zr}$	$^{95}\text{Zr}$	0,957566	0,87700	$^{174}\text{Yb}$	$^{175}\text{Yb}$	0,357880	0,221067
$^{96}\text{Zr}$	$^{97}\text{Zr}$	0,999115	0,997943	$^{176}\text{Yb}$	$^{177}\text{Yb}$	0,908488	0,809688
$^{95}\text{Nb}$	$^{94m}\text{Nb}$	0,969700	0,928380	$^{175}\text{Lu}$	$^{176m}\text{Lu}$	0,996032	0,991774
$^{98}\text{Mo}$	$^{99}\text{Mo}$	0,995883	0,990853	$^{174}\text{Hf}$	$^{175}\text{Hf}$	0,759887	0,609407
$^{100}\text{Mo}$	$^{101}\text{Mo}$	0,988295	0,970848	$^{179}\text{Hf}$	$^{180m}\text{Hf}$	0,990364	0,980132
$^{96}\text{Ru}$	$^{97}\text{Ru}$	0,991702	0,978811	$^{180}\text{Hf}$	$^{181}\text{Hf}$	0,913130	0,837096
$^{102}\text{Ru}$	$^{103}\text{Ru}$	0,938892	0,877634	$^{181}\text{Ta}$	$^{182m}\text{Ta}$	0,997216	0,992780
$^{104}\text{Ru}$	$^{105}\text{Ru}$	0,982706	0,958954	$^{186}\text{W}$	$^{187}\text{W}$	0,988597	0,977570
$^{105}\text{Rh}$	$^{104m}\text{Rh}$	1,29702	1,11028	$^{185}\text{Re}$	$^{186}\text{Re}$	1,014020	0,998171
$^{108}\text{Pd}$	$^{109m}\text{Pd}$	0,993450	0,987460	$^{187}\text{Re}$	$^{188m}\text{Re}$	0,958039	0,921987
$^{110}\text{Pd}$	$^{111m}\text{Pd}$	0,989017	0,971347	$^{190}\text{Os}$	$^{191m}\text{Os}$	0,891403	0,800890
$^{107}\text{Ag}$	$^{108}\text{Ag}$	0,934276	0,880183	$^{192}\text{Os}$	$^{193}\text{Os}$	0,908020	0,831151
$^{109}\text{Ag}$	$^{110m}\text{Ag}$	1,00012	0,990589	$^{193}\text{Ir}$	$^{194}\text{Ir}$	1,049670	1,015320
$^{114}\text{Cd}$	$^{115}\text{Cd}$	0,994496	0,988007	$^{198}\text{Pt}$	$^{199m}\text{Pt}$	0,987525	0,974821
$^{113}\text{In}$	$^{114m}\text{In}$	0,999628	0,993641	$^{197}\text{Au}$	$^{198}\text{Au}$	1,001300	0,990335
$^{115}\text{In}$	$^{116m}\text{In}$	1,07891	1,03788	$^{196}\text{Hg}$	$^{197m}\text{Hg}$	0,493779	-0,32989
$^{112}\text{Sn}$	$^{113m}\text{Sn}$	0,995628	0,991087	$^{238}\text{U}$	$^{239}\text{U}$	1,0004	0,99725

## II. BASE OF APPROXIMATION

In the approximation of J. OP De BEEK, it is good for the nuclides having  $Q_{0i} > 1$ , but is not for the nuclides with  $Q_{0i} < 1$ . Indeed, in case of  $^{45}\text{Sc}(n,\gamma)^{46}\text{Sc}$  with  $Q_0 = 0.44$ ,  $\bar{E}_r = 5130$  eV, if  $\alpha = 0.1$ , the value of  $Q_0(\alpha)$  from Eq. (4) of J. OP De BEEK is 0.18, but from Eq. (3) it is 0.383. The difference is a factor of 2. If  $\alpha = 0.2$ ,  $Q_0(\alpha)$  is 0.08 and 0.345 from Eq. (4) and Eq. (3), respectively. The difference is more than four times (!). Correspondingly, some results in [6] are not accurate and needed to be discussed.

As we know,  $\alpha$  value is smaller than unity in absolute value. In practice, in irradiation channels of reactor, absolute value of  $\alpha$  is less than 0.2 (in most cases,  $|\alpha| < 0.1$  and this condition is satisfactory in reactor core). For this reason,  $Q_{0i}(\alpha)$  from Eq. (3) can be written in a simpler approximative expression. Due to  $|\alpha| \ll 1$ , in [7,8], we suggest substituting  $Q_{0,i}(\alpha)$  from Eq. (3) by the following approximated formula:

$$Q_{0i}(\alpha) \approx Q_{0i}(\bar{E}_{ri})^{-a_i \alpha} \text{ or}$$

$$Q_{0i}(\alpha) \approx Q_{0i} \exp(-a_i (\ln \bar{E}_{ri}) \alpha) \quad (5)$$

Where  $a_i$  is constant for each nuclide and determined by fitting the values of  $Q_{0i}(\alpha)$ , which are calculated from Eq. (3) in range  $|\alpha| \leq 0.2$ , then fitting according to function (5) (see [7,8]). Note that,  $a_i$  of each nuclide depends on the sign of  $\alpha$ . The values of  $a_i$  for the interested nuclides in NAA are given in Table 1. Seeing the Eq. (5), it differs to Eq. (4) of J. OP De BEEK by a correctional coefficient  $a_i$ . However, it can be used good for all nuclides with uncertainties of the calculated  $Q_{0,i}(\alpha)$  less than about 5% for the nuclides having  $Q_{0i} < 1$  and less than about 2% for  $Q_{0i} > 1$  with  $|\alpha| \leq 0.2$ . Indeed, we carried out a survey of the ratios of  $Q_{0i}(\alpha)$  calculated from Eq.(5) (in this work) and Eq. (4) (of J. OP De BEEK) to Eq. (3) (accurate expression) for  $Q_{0i}$  from 0.44 ( $^{46}\text{Sc}$ ) to 248 ( $^{97}\text{Zr}$ ) with  $\alpha = -0.1$  and the results are presented in Fig. 1.

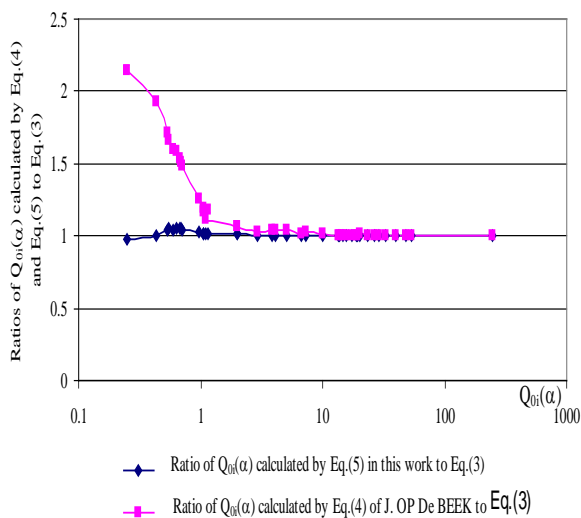


Fig 1. Survey of the ratios of  $Q_{0i}(\alpha)$  calculated from Eq.(5) (in this work) and Eq. (4) (of J. OP De BEEK) to Eq. (3) (accurate expression) for different  $Q_{0i}$  with  $\alpha = -0.1$

Clearly, the approximated expression in this work is better than one of J. OP De BEEK. Moreover, the calculated  $Q_{0i}(\alpha)$  from three expression Eq.(3), Eq. (4) and Eq. (5) for  $^{45}\text{Sc}(n,\gamma)^{46}\text{Sc}$  presented in Table 2 and the another nuclides presented in papers [7,8] also confirm the above conclusion.

From Table 1, it shows that coefficients  $a_i$  of nuclides having  $Q_{0i} > 1$  are close to unity, but  $a_i$  of the nuclides having  $Q_{0i} < 1$  differs more than unity. Therefore, the approximation of Eq. (4) in paper [6] is only acceptable for the nuclides having  $Q_{0i} > 1$ , but for the nuclides having  $Q_{0i} < 1$ , it is not reliable.

In this work, we use the approximation expression; Eq. (5), to evaluate influence of the uncertainties of  $\alpha$ ,  $f$  and  $\bar{E}_{ri}$  on the final element concentration in  $k_0$ -method in the channels; 7-1, neutron trap of Dalat reactor (Vietnam) and channel 17 of THETIS reactor (Belgium) for the nuclides;  $^{45}\text{Sc}$ ,  $^{59}\text{Co}$ ,  $^{94}\text{Zr}$ ,  $^{186}\text{W}$ ,  $^{197}\text{Au}$ ,  $^{98}\text{Mo}$ ,  $^{96}\text{Zr}$ . We choose these nuclides, because they differ considerably in  $Q_{0i}$  and  $\bar{E}_{ri}$  values. The numerical data of concerning isotopes and irradiation channels used in this work are summarized in Tables 3 and 4.

TABLE II THE VALUES OF  $Q_{0i}(\alpha)$  CALCULATED FROM THREE EXPRESSION Eq.(3), Eq. (4) AND Eq. (5) WITH  $\alpha$  IN INTERVAL  $[-0.2, 0.2]$  FOR  $^{45}\text{Sc}(n,\gamma)^{46}\text{Sc}$

Value of $\alpha$	$Q_{0i}(\alpha)$ from Eq.3	$Q_{0i}(\alpha)$ from Eq.5	$Q_{0i}(\alpha)$ from Eq.4	Value of $\alpha$	$Q_{0i}(\alpha)$ from Eq.3	$Q_{0i}(\alpha)$ from Eq.5	$Q_{0i}(\alpha)$ from Eq.4
-0.20	0,7072	0,6850	2,429	0,02	0,4263	0,4287	0,371
-0.18	0,6629	0,6554	2,048	0,04	0,4139	0,4178	0,313
-0.16	0,6242	0,6270	1,726	0,06	0,4026	0,4071	0,264
-0.14	0,5905	0,5998	1,455	0,08	0,3923	0,3967	0,222
-0.12	0,5607	0,5738	1,227	0,10	0,3828	0,3865	0,187
-0.10	0,5345	0,5490	1,034	0,12	0,3741	0,3767	0,158
-0.08	0,5112	0,5252	0,872	0,14	0,3661	0,3671	0,133
-0.06	0,4905	0,5025	0,7346	0,16	0,3587	0,3577	0,112
-0.04	0,4718	0,4807	0,619	0,18	0,3518	0,3486	0,095
-0.02	0,4551	0,4599	0,522	0,20	0,3455	0,3396	0,080

Notice: Eq.3: true expression Eq.5: Expression in this work Eq.4: Expression in [6] of J. Op De Beek

TABLE III CHARACTERISTICS OF ISOTOPES USED IN THE CALCULATION IN THIS WORK

Nuclide	$Q_0 = I_0/\sigma_0$	$\bar{E}_r$ (eV)
$^{45}\text{Sc}$	0,44	5130
$^{59}\text{Co}$	1,993	136
$^{94}\text{Zr}$	5,05	6260
$^{186}\text{W}$	13,7	20,5
$^{197}\text{Au}$	15,7	5,65
$^{98}\text{Mo}$	53,1	241
$^{96}\text{Zr}$	248	338

### III. RESULTS AND DISCUSSION

The absolute uncertainty in  $\rho$  can be calculated from the uncertainties of the variables (denoted  $x_j$ ) which determine  $\rho$  in Eq. (1):

$$s_p = \sqrt{\sum_j s_{x_j}^2 \left( \frac{\partial \rho}{\partial x_j} \right)^2} \quad (6)$$

where  $\partial \rho / \partial x_j$  are the corresponding partial derivatives.

TABLE IV CHARACTERISTICS OF IRRADIATION CHANNELS

Channel	$\alpha$	$f$
Channel 7-1 (Dalat reactor)	$-0.044 \pm 0.004$	$14.2 \pm 0.5$
Neutron trap (Dalat reactor)	$-0.031 \pm 0.004$	$33.0 \pm 0.5$
Channel 17 (Thetis reactor)[9]	-0.028	15.0

According to the customary error propagation theory, the error propagation functions can be written as:

$$Z_p(x_j) = \left| \frac{\partial \rho}{\partial x_j} \right| \left( \frac{x_j}{\rho} \right) = \left| \frac{\partial \rho}{\partial x_j} \frac{x_j}{\rho} \right| \quad (7)$$

and relative error is:

$$s_p(x_j) = Z_p(x_j) \frac{\Delta x_j}{x_j} \quad (8)$$

#### 1) Influence of Uncertainty of $\bar{E}_{ri}$ on NAA Results

From Eq. (8), the uncertainty of the concentration ( $\rho$ ) in  $k_0$ -method due to the uncertainties of the effective resonance energies can be written by:

$$\frac{\Delta \rho}{\rho} \Big|_{\bar{E}_{ri}} = Z_p(\bar{E}_{ri}) \frac{\Delta \bar{E}_{ri}}{\bar{E}_{ri}} \quad (9)$$

Using Eq.(7) for the effective resonance energy of the nuclide  $i$ , we obtain:

$$Z_p(\bar{E}_{ri}) = \left| \alpha \left( \frac{a_i Q_{0i}}{Q_{0i} + f(\bar{E}_{ri})^{a_i \alpha}} \right) \right| \quad (10)$$

The values of calculated  $Z_p(\bar{E}_{ri})$  for choosen nuclides are presented in Table 5. The effect of the effective resonance energy on NAA result include the uncertainties of the effective resonance energies of analytical and comparator nuclides. In this case, Au used as comparator with  $\bar{E}_{Au}$  of 5,65 eV and uncertainty of 7.1% from [5], the contribution of the uncertainty of  $\bar{E}_{Au}$  to the error of NAA result in channels 7-1, neutron trap of Dalat reactor and channel 17 of THETIS reactor is 0,17%, 0,077% and 0,13%, respectively. Clearly, the effect of the uncertainty of the effective resonance energy of Au is negligible and can be overlooked in the evaluation.

TABLE V CALCULATED RESULTS OF  $Z_p(\bar{E}_{ri})$  FOR CHOOSEN NUCLIDES

Nuclide	7-1 channel (Dalat reactor)	Neutron trap (Dalat reactor)	Channel 17 (THETIS reactor)
<sup>45</sup> Sc	0,000316	0,000102	0,000185
<sup>59</sup> Co	0,005765	0,001939	0,00329
<sup>94</sup> Zr	0,01430	0,005304	0,008004
<sup>185</sup> W	0,02278	0,01025	0,01379
<sup>197</sup> Au	0,02396	0,01106	0,01467
<sup>98</sup> Mo	0,03620	0,02163	0,02244
<sup>96</sup> Zr	0,04601	0,03065	0,02671

The analysis for 94 nuclides used in NAA showed that the uncertainties of their effective resonance energy are from 0 to 20%, except <sup>75</sup>As (34%) [4]. In this measure, we are able to realize that the effect of them on NAA result is also negligible. For example, <sup>45</sup>Sc ( $\bar{E}_r = 5130$  eV,  $\Delta \bar{E}_r = 17\%$ ) and <sup>95</sup>Zr ( $\bar{E}_r = 338$  eV,  $\Delta \bar{E}_r = 2.1\%$ ), the contribution of the uncertainty of the effective resonance energy to the error of NAA result in three above channels is less than 0.01% for <sup>45</sup>Sc and 0.1% for <sup>95</sup>Zr.

In epicadmium neutron activation analysis (ENAA), the function in Eq. (10) should be omitted. The error propagation function of  $\bar{E}_{ri}$  can be written:

$$Z_p(\bar{E}_{ri}) = |\alpha a_i| \quad (11)$$

The calculated results of  $Z_p(\bar{E}_{ri})$  for for the nuclides; <sup>45</sup>Sc, <sup>59</sup>Co, <sup>94</sup>Zr, <sup>186</sup>W, <sup>197</sup>Au, <sup>98</sup>Mo, <sup>96</sup>Zr in ENAA are carried in Table 6. In this case, the error propagation function is higher than in the one of irradiation without cadmium. Generally speaking,  $a_i < 1$  and if  $\alpha \ll 1$ , the contribution of  $\Delta \bar{E}_{ri}$  to the error of NAA result for almost

analytical nuclides is less than 1% and can be omitted in the calculation.

TABLE VI CALCULATED RESULTS OF  $Z_p(\bar{E}_{ri})$  FOR THE NUCLIDES IN ENAA

Nuclide	7-1 channel (Dalat reactor)	Neutron trap (Dalat reactor)	Channel 17 (THETIS reactor)
<sup>45</sup> Sc	0,00991	0,00743	0,00631
<sup>59</sup> Co	0,03958	0,02969	0,02519
<sup>94</sup> Zr	0,04213	0,03160	0,02681
<sup>185</sup> W	0,04350	0,03262	0,02768
<sup>197</sup> Au	0,04406	0,03304	0,02803
<sup>98</sup> Mo	0,04382	0,03286	0,02788
<sup>96</sup> Zr	0,04396	0,03329	0,02797

## 2) Influence of Uncertainty of $\alpha$ on NAA Results

Also from Eq. (8), the uncertainty of  $\rho$  due to the uncertainty of  $\alpha$  can be written:

$$\frac{\Delta \rho}{\rho} \Big|_{\alpha} = Z_p(\alpha) \frac{\Delta \alpha}{\alpha} \quad (12)$$

and error propagation function of  $\alpha$ :

$$Z_p(\alpha) = \left| -\alpha \left( \frac{a_i^* Q_{0i}^* \ln(\bar{E}_{ri}^*)}{Q_{0i}^* + f(\bar{E}_{ri}^*)^{a_i^* \alpha}} - \frac{a_i Q_{0i} \ln(\bar{E}_{ri})}{Q_{0i} + f(\bar{E}_{ri})^{a_i \alpha}} \right) \right| \quad (13)$$

The values of the error propagation function of  $\alpha$  in the channels; 7-1 and neutron trap of Dalat reactor (Vietnam) and channel 17 of THETIS reactor (Belgium) for the nuclides; <sup>45</sup>Sc, <sup>59</sup>Co, <sup>94</sup>Zr, <sup>186</sup>W, <sup>197</sup>Au, <sup>98</sup>Mo, <sup>96</sup>Zr were shown in Table 7. From Table 7, for the nuclides having  $Q_0 < Q_{0Au}$  in three these channels, the contribution of the uncertainty of  $\alpha$  to the error of NAA result is not significant, about less than 1%. But for nuclides having  $Q_0 \gg Q_{0Au}$ , this effect is noticeable. For instance, in channel 7-1 of Dalat reactor ( $\alpha = -0.044$ ,  $\Delta \alpha = 12\%$  [7,8]), the contribution of the uncertainty of  $\alpha$  on the error of result of <sup>45</sup>Sc ( $Q_0 = 0.44$ ) is 0.42%, but for <sup>99</sup>Mo and <sup>96</sup>Zr is 1,36% and 2.4%, respectively. As a comment, for RNAA using <sup>197</sup>Au comparator, the systematic effect for  $\alpha$  value up to 0.1 is practically negligible for all nuclides with a low enough  $Q_0$  value (e.g. <sup>45</sup>Sc, <sup>59</sup>Co, <sup>58</sup>Fe, etc.). On the other hand, for nuclides with a relatively large  $Q_0$  value, a correction for the  $\alpha$  effect becomes really necessary. To reduce the  $\alpha$  effect, it is either to develop more accurate and precise techniques for  $\alpha$  determination or to choose the irradiation channels with the  $\alpha$  value low enough.

In the case of the epicadmium neutron activation, Eq. (13) can be changed into:

$$Z_p(\alpha) = \left| -\alpha \left( a_i^* \ln(\bar{E}_{ri}^*) - a_i \ln(\bar{E}_{ri}) \right) \right| \quad (14)$$

The values of the error propagation of  $\alpha$  in this case were carried in Table 8. In this case, it clearly shows the inaccuracy of the approximation expression in [6] (Eq. (4) in this report). Really, according to Eq. (4), the error propagation function of  $\alpha$  in the irradiation with cadmium can be written:

$$Z_p(\alpha) = \left| -\alpha \left( \ln(\bar{E}_{ri}^*) - \ln(\bar{E}_{ri}) \right) \right| \quad (15)$$

TABLE VII

CALCULATED RESULTS OF  $Z_p(\alpha)$  FOR CHOSEN NUCLIDES

Nuclide	7-1 channel (Dalat reactor)	Neutron trap (Dalat reactor)	Channel 17 (THETIS reactor)
<sup>45</sup> Sc	0,03571	0,01684	0,02241
<sup>59</sup> Co	0,01755	0,01022	0,01055
<sup>94</sup> Zr	0,03379	0,01083	0,02489
<sup>185</sup> W	0,02154	0,00916	0,01390
<sup>197</sup> Au	0,000	0,000	0,000
<sup>98</sup> Mo	0,11340	0,07016	0,07294
<sup>96</sup> Zr	0,19924	0,14750	0,12751

TABLE VIII

CALCULATED RESULTS OF  $Z_p(\alpha)$  FOR THE NUCLIDES IN ENAA

Nuclide	7-1 channel (Dalat reactor)	Neutron trap (Dalat reactor)	Channel 17 (THETIS reactor)
<sup>45</sup> Sc	0,00835	0,00626	0,00531
<sup>59</sup> Co	0,11817	0,08863	0,07520
<sup>94</sup> Zr	0,2920	0,2190	0,18584
<sup>186</sup> W	0,05509	0,04132	0,03506
<sup>197</sup> Au	0,000	0,000	0,000
<sup>98</sup> Mo	0,1640	0,1230	0,10439
<sup>96</sup> Zr	0,17969	0,13477	0,11435

Eq. (15) is different to Eq. (14) by the correctional coefficients  $a_i$ . However, the value of the error propagation function in channel 7-1 of Dalat reactor, for <sup>45</sup>Sc is 0.0083 from Eq.(14) and 0.2997 from Eq. (15). If the uncertainty of  $\alpha$  in experiment is 100%, the contribution of uncertainty of  $\alpha$  on NAA result is 0.83% and 29.97%, respectively. It differs by a factor of 30 (!). Similarly, in channel 17 of Thetis reactor, the error propagation function for <sup>45</sup>Sc is 0.0053 and 0.1907. The difference is huge. This comment is also correct for nuclides having  $Q_0 < 1$ . It once more confirms that the approximation expression in papper [6] is not good for nuclides having  $Q_0 < 1$ . From Eq. (13) or Eq. (14), we easily estimate the influence of  $\alpha$  on NAA results, if we know uncertainty of  $\alpha$  in the irradiation channel. However, for ENAA (epicadmium neutron activation analysis) the situation is much more dramatic, especially for nuclides with low  $Q_0$  value.

### 3) Influence of Uncertainty of $f$ on NAA Results

The error propagation function  $Z_p(f)$  can be written:

$$Z_p(f) = \left| -f \frac{Q_{0i}^* (\overline{E_{ri}})^{a_i \alpha} - Q_{0i} (\overline{E_{ri}}^*)^{a_i \alpha}}{(Q_{0i}^* + f(\overline{E_{ri}}^*)^{a_i \alpha})(Q_{0i} + f(\overline{E_{ri}})^{a_i \alpha})} \right| \quad (16)$$

The values of the error propagation function of  $f$  in the channels; 7-1 and neutron trap of Dalat reactor and channel 17 of THETIS reactor for the nuclides; <sup>45</sup>Sc, <sup>59</sup>Co, <sup>94</sup>Zr, <sup>186</sup>W, <sup>197</sup>Au, <sup>98</sup>Mo, <sup>96</sup>Zr were carried in Table 9. The uncertainty of  $f$  contributes on the error of NAA results is:

$$\frac{\Delta \rho}{\rho} = Z_p(f) \frac{\Delta f}{f} \quad (17)$$

Generally seeing, the uncertainty of  $f$  in experiment is about less than 4%, therefor, from Table 9, the contribution of the uncertainty of  $f$  on the error of NAA result is about less than 2%.

TABLE IX

CALCULATED RESULTS OF  $Z_p(f)$  FOR THE CHOSEN NUCLIDES

Nuclide	Channel 7-1 (Dalat reactor)	Neutron trap (Dalat reactor)	Channel 17 (THETIS reactor)
<sup>45</sup> Sc	0,512	0,321	0,494
<sup>59</sup> Co	0,398	0,2697	0,393
<sup>94</sup> Zr	0,204	0,167	0,225
<sup>185</sup> W	0,0202	0,0208	0,0252
<sup>197</sup> Au	0,0	0,0	0,0
<sup>98</sup> Mo	0,282	0,323	0,281
<sup>96</sup> Zr	0,413	0,566	0,428

### 4) Collective Influence of Uncertainties of $\alpha$ , $\overline{E_{ri}}$ and $f$ on NAA Results

In view of the above, we can estimate the influence of the uncertainties of  $\alpha$ ,  $\overline{E_{ri}}$  and  $f$  on final NAA results. The contribution of these parameters on the errors of the analysis results is written as:

$$\frac{\Delta \rho}{\rho} \Big|_{\alpha, \overline{E_{ri}}, f} = \sqrt{\left( \frac{\Delta \rho}{\rho} \Big|_{\alpha} \right)^2 + \left( \frac{\Delta \rho}{\rho} \Big|_{\overline{E_{ri}}} \right)^2 + \left( \frac{\Delta \rho}{\rho} \Big|_f \right)^2} \quad (18)$$

However, as discussion above, the  $\overline{E_r}$  effect is negligible and can be omitted in Eq. (18). Thus, the contribution on error of NAA results in this case is primarily due to the uncertainties of  $\alpha$  and  $f$ . Finally, as well as estimation above, this overall contribution of  $\alpha$  and  $f$  is about 2% on the error of NAA results. It was also confirmed by actual analysis.

## IV. CONCLUSION

For  $\alpha$  in the irradiation position relatively small ( $\alpha \ll 1$ ), Eq.(5) is a good approximation to estimate influence of the neutron flux characteristics on NAA result using the  $k_0$ -standardization method. From this approximative expression, the error propagation functions of the parameters were presented. They can be used for the estimation of the errors on NAA due to the uncertainty of the neutron flux characteristic parameters at the irradiation position. From the results of this report, it was also confirmed that the approximation in papper [6] is only acceptable for the nuclides having  $Q_{0i} > 1$ , but not for the nuclides having  $Q_{0i} < 1$ .

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